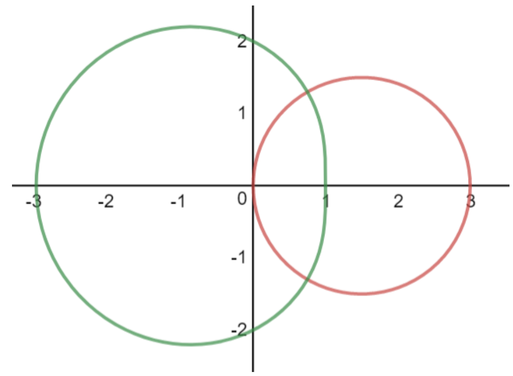
AP Calculus BC – McGlone
Section 10.6 – Polar Area Multiple Curves

Find the indicated information. Some can be set up without your calculator. Integrate with your calculator.

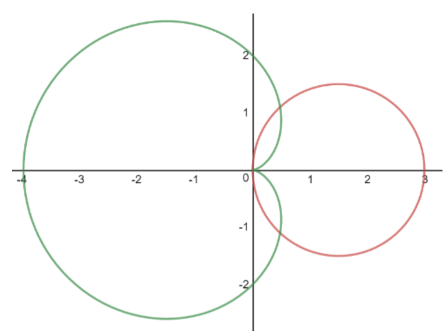
1. For the polar curves $r = 3cos\theta$ and $r = 2 - cos\theta$:

   a. Find the values of θ where the curves intersect (no calculator.)
   b. Find the area inside the circle but outside the limaçon.
   c. Find the area shared by the curves.

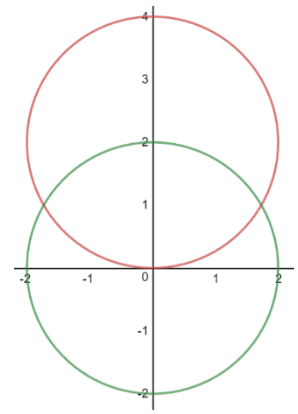


2. For the polar curves $r = 3cos\theta$ and $r = 2 - 2cos\theta$:

   a. Find the values of θ where the curves intersect (calculator.)
   b. Find the area shared by the curves.
   c. Find the area outside the circle but inside the cardioid in QI.

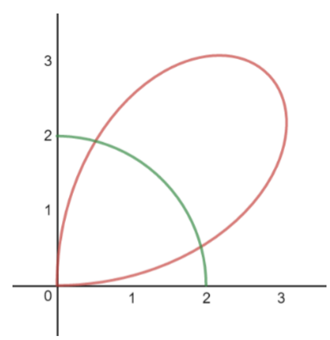


3. For the polar curves $r = 4sin\theta$ and $r = 2$:



    a. Find the values of θ where the curves intersect (no calculator.)
    b. Find the area shared by the curves.
    c. Find the area inside $r = 4sin\theta$ but outside $r = 2$ with a single integral expression.

4. For the polar curves $r = 4\sin(2\theta)$ and $r = 2$:



    a. Find the values of θ where the curves interest in QI only (calculator.)
    b. Find the area outside of the circle but inside the rose in QI.
    c. Find the area shared by the curves in QI only without using subtraction.

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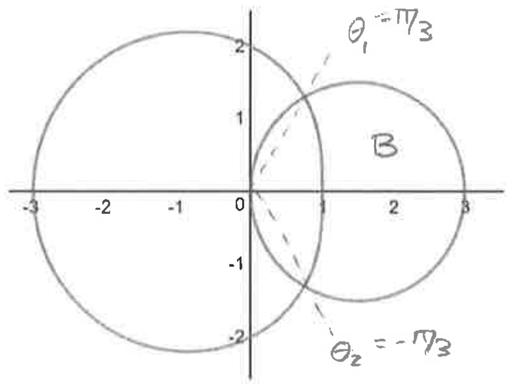
Find the indicated information. Some can be set up without your calculator. Integrate with your calculator.

1. For the polar curves $\overset{r_1}{r} = 3cos\theta$ and $\overset{r_2}{r} = 2 - cos\theta$:



   a. Find the values of θ where the curves intersect (no calculator.)
   b. Find the area inside the circle but outside the limaçon.
   c. Find the area shared by the curves.

a) $3\cos\theta = 2 - \cos\theta$

$4\cos\theta = 2$

$\cos\theta = 1/2$

$\theta_1 = \pi/3$

$\theta_2 = -\pi/3$

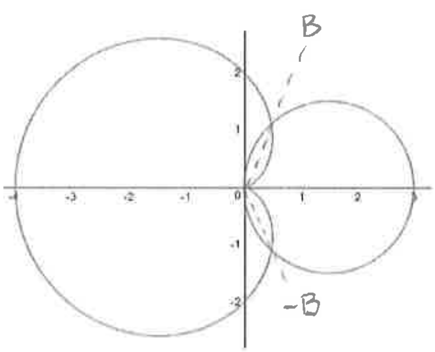
b) $2 \cdot \frac{1}{2} \int_0^{\pi/3} (3\cos\theta)^2 - (2-\cos\theta)^2 \, d\theta$

$= 5.196$

c) $2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{3}} (2-\cos\theta)^2 \, d\theta + 2 \cdot \frac{1}{2} \int_{\pi/3}^{\pi/2} (3\cos\theta)^2 \, d\theta$

$= 1.872$

2. For the polar curves $r_1 = 3cos\theta$ and $r_2 = 2 - 2cos\theta$:



   a. Find the values of θ where the curves intersect (calculator.)
   b. Find the area shared by the curves.
   c. Find the area outside the circle but inside the cardioid in QI.

a) $3\cos\theta = 2 - 2\cos\theta$

$5\cos\theta = 2$

$\cos\theta = 2/5$

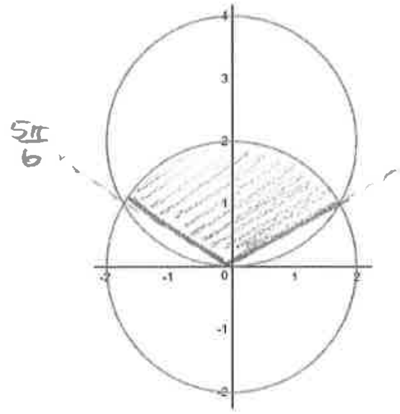
$\theta \approx 1.159... = B$

b) $2 \cdot \frac{1}{2} \int_0^B (2-2\cos\theta)^2 d\theta + 2 \cdot \frac{1}{2} \int_B^{\pi/2} (3\cos\theta)^2 d\theta = .5588...$

c) $\frac{1}{2} \int_B^{\frac{\pi}{2}} (2-2\cos\theta)^2 - (3\cos\theta)^2 \, d\theta = .4329...$

3. For the polar curves $r_1 = 4sin\theta$ and $r_2 = 2$:

a. Find the values of θ where the curves intersect (no calculator.)
b. Find the area shared by the curves.
c. Find the area inside $r = 4sin\theta$ but outside $r = 2$ with a single integral expression.



a) $4\sin\theta = 2$

$\sin\theta = 1/2$

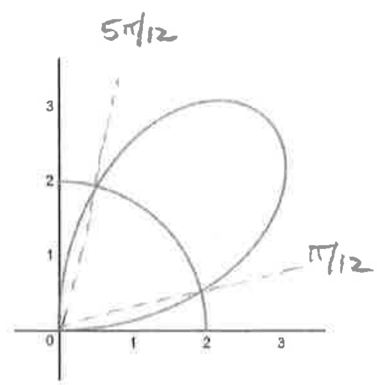
$\theta = \pi/6$

b) $2\cdot\frac{1}{2}\int_0^{\pi/6}(4\sin\theta)^2 d\theta + \frac{1}{3}\pi(2)^2 = 4.913$

c) $2\cdot\frac{1}{2}\int_{\pi/6}^{\pi/2}(4\sin\theta)^2 - 2^2\, d\theta = 7.6528$

4. For the polar curves $r = 4\sin(2\theta)$ and $r = 2$:

a. Find the values of θ where the curves interest in QI only (calculator.)
b. Find the area outside of the circle but inside the rose in QI.
c. Find the area shared by the curves in QI only without using subtraction.



a) $4\sin 2\theta = 2$

$\sin 2\theta = 1/2$

$2\theta = \pi/6$

$\theta = \pi/12 \approx .2617\ldots = B$

b) $\frac{1}{2}\int_{\pi/12}^{5\pi/12}(4\sin(2\theta))^2 - 4\, d\theta = 3.826$

c) $2\cdot\frac{1}{2}\int_0^{\pi/12}(4\sin 2\theta)^2 + \frac{1}{2}\int_{\pi/12}^{5\pi/12} 2^2\, d\theta = 2.4567\ldots$